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# On Rough $I-$ Core of Triple Sequence Spaces by Metric

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**Abstract.** We introduce and study some basic properties of rough $I-$ convergent of triple sequence spaces and also study the set of all rough $I-$ limits of a triple sequence spaces and relation between analytic ness and rough $I-$ core of a triple sequence spaces.

**Keywords.** Triple sequences, rough convergence, closed and convex, cluster points and rough limit points.

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## 1. Introduction

The idea of rough convergence was first introduced by Phu [9-11] in finite dimensional normed spaces. He showed that the set $LIM_x^r$ is bounded, closed and convex; and he introduced the notion of rough Cauchy sequence. He also investigated the relations between rough convergence and other convergence types and the dependence of $LIM_x^r$ on the roughness of degree $r$. Aytar [1] studied of rough statistical convergence and defined the set of rough statistical limit points of a sequence and obtained two statistical convergence criteria associated with this set and prove that this set is closed and convex. Also, Aytar [2] studied that the $r-$ limit set of the sequence is equal to intersection of these sets and that $r-$ core of the sequence is equal to the union of these sets. D$\ddot{u}$ndar and Cakan [8] investigated of rough ideal convergence and defined the set of rough ideal limit points of a sequence The notion of $I-$ convergence of a triple sequence spaces which is based on the structure of the ideal $I$ of subsets of $\mathbb{N}\times\mathbb{N}\times\mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers, is a natural generalization of the notion of convergence and statistical convergence. In this paper we investigate some basic properties of rough $I-$ convergence of a triple sequence spaces in three dimensional matrix spaces which are not earlier. We study the set of all rough $I-$ limits of a triple sequence spaces and also the relation between analytic ness and rough $I-$ core of a triple sequence spaces. Let $K$ be a subset of the set of positive integers $\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ and let us denote
the set $K_{ik\ell} = \{(m,n,k) \in K : m \geq i, n \leq j, k \leq \ell\}$. Then the natural density of $K$ is given by

$$\delta(K) = lim_{i,j,\ell\to\infty} \frac{|K_{ij\ell}|}{ij\ell},$$

where $|K_{ij\ell}|$ denotes the number of elements in $K_{ij\ell}$. Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers , $\chi_A-$ the characteristic function of $A \subset \mathbb{N}, \mathbb{R}$ the set of all real numbers. A subset $A$ of $\mathbb{N}$ is said to have asymptotic density $d(A)$ if

$$d(A) = lim_{ij\ell\to\infty} \frac{1}{ij\ell} \sum_{m=1}^{i} \sum_{n=1}^{j} \sum_{k=1}^{\ell} \chi_A(K).$$

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A triple sequence (real or complex) can be defined as a function $x:\mathbb{N}\times\mathbb{N}\times\mathbb{N}\to\mathbb{R}\,(\mathbb{C})$, where $\mathbb{N},\mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by *Sahiner et al. [12,13]*, *Esi et al. [3-5]*, *Datta et al. [6]*,*Subramanian and Esi [14]*, *Debnath et al. [7]* and many others.
A triple sequence $x=(x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k}\left|x_{mnk}\right|^{\frac{1}{m+n+k}}<\infty.$$

The space of all triple analytic sequences are usually denoted by $\Lambda^3$.

## 2. Definitions and Preliminaries

Throughout the paper $\mathbb{R}^m\times\mathbb{R}^n\times\mathbb{R}^k=\mathbb{R}^3$ denotes the real three dimensional case with the metric. Consider a triple sequence spaces $x=(x_{mnk})$ such that $x_{mnk}\in\mathbb{R}^3=\mathbb{R}^m\times\mathbb{R}^n\times\mathbb{R}^k;m,n,k\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}$. The following defintion are obtained:

### 2.1. Definition

A triple sequence spaces $x=(x_{mnk})$ of real numbers is said to be statistically convergent to $L\in\mathbb{R}^3$ if for any $\varepsilon>0$ we have $d\,(A\,(\varepsilon))=0$, where

$$A\,(\varepsilon)=\left\{(m,n,k)\in\mathbb{N}^3:|x_{mnk}-L|\geq\varepsilon\right\}.$$

### 2.2. Definition

A triple sequence spaces $x=(x_{mnk})$ is said to be statistically convergent to $L\in\mathbb{R}^3$, written as $st-limx=L$, provided that the set

$$\left\{(m,n,k)\in\mathbb{N}^3:|x_{mnk}-L|\geq\varepsilon\right\},$$

has natural density zero for every $\varepsilon>0$. In this case, $L$ is called the statistical limit of the sequence $x$.

### 2.3. Definition

Let $x=(x_{mnk})_{m,n,k\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ be a triple sequence spaces in a metric space $(X,|.,.|)$ and $r$ be a nonnegative real number. Atriple sequence spaces $x=(x_{mnk})$ is said to be $r-$convergent to $L\in X$, denoted by $x\to^r L$, if for any $\varepsilon>0$ there exists $N_\varepsilon\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ such that for all $m,n,k\geq N_\varepsilon$ we have

$$|x_{mnk}-L|<r+\varepsilon$$

In this case $L$ is called an $r-$ limit of $x$.

### 2.4. Remark

We consider $r-$ limit set $x$ which is denoted by $LIM_x^r$ and is defined by

$$LIM_x^r=\{L\in X:x\to^r L\}.$$

### 2.5. Definition

A triple sequence spaces $x=(x_{mnk})$ is said to be $r-$ convergent if $LIM_x^r\neq\phi$ and $r$ is called a rough convergence degree of $x$. If $r=0$ then it is ordinary convergence of triple sequence spaces.

### 2.6. Definition

Let $x = (x_{mnk})$ be a triple sequence spaces in a metric space $(X, |.,.|)$ and $r$ be a nonnegative real number is said to be $r-$ statistically convergent to $L$, denoted by $x \rightarrow^{r-st_3} L$, if for any $\varepsilon > 0$ we have $d(A(\varepsilon)) = 0$, where

$$A(\varepsilon) = \{(m,n,k) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{mnk} - L| \geq r + \varepsilon\}.$$

In this case $L$ is called $r-$ statistical limit of $x$. If $r = 0$ then it is ordinary statistical convergent of triple sequence spaces.

### 2.7. Definition

A class $I$ of subsets of a nonempty set $X$ is said to be an ideal in $X$ provided
(i) $\phi \in I$
(ii) $A, B \in I$ implies $A \bigcup B \in I$.
(iii) $A \in I, B \subset A$ implies $B \in I$.
$I$ is called a nontrivial ideal if $X \notin I$.

### 2.8. Definition

A nonempty class $F$ of subsets of a nonempty set $X$ is said to be a filter in $X$. Provided
(i) $\phi \in F$.
(ii) $A, B \in F$ implies $A \bigcap B \in F$.
(iii) $A \in F, A \subset B$ implies $B \in F$.

### 2.9. Definition

$I$ is a non trivial ideal in $X$, $X \neq \phi$, then the class

$$F(I) = \{M \subset X : M = X \backslash A \ for\ some\ A \in I\}$$

is a filter on $X$, called the filter associated with $I$.

### 2.10. Definition

A non trivial ideal $I$ in $X$ is called admissible if $\{x\} \in I$ for each $x \in X$.

### 2.11. Note

If $I$ is an admissible ideal, then usual convergence in $X$ implies $I$ convergence in $X$.

### 2.12. Remark

If $I$ is an admissible ideal, then usual rough convergence implies rough $I-$ convergence.

### 2.13. Definition

Let $x = (x_{mnk})$ be a triple sequence in a metric space $(X, |.,.|)$ and $r$ be a nonnegative real number is said to be rough ideal convergent or $rI-$ convergent to $L$, denoted by $x \rightarrow^{rI} L$, if for any $\varepsilon > 0$ we have

$$\{(m,n,k) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{mnk} - L| \geq r + \varepsilon\} \in I.$$

In this case $L$ is called $rI-$ limit of $x$ and a triple sequence spaces $x = (x_{mnk})$ is called rough $I-$ convergent to $L$ with $r$ as roughness of degree. If $r = 0$ then it is ordinary $I-$ convergent.

### 2.14. Note

Generally, a triple sequence spaces $y = (y_{mnk})$ is not $I-$ convergent in usual sense and $|x_{mnk} - y_{mnk}| \leq r$ for all $(m,n,k) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ or

$$\{(m,n,k) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{mnk} - y_{mnk}| \geq r\} \in I.$$

for some $r > 0$. Then the triple sequence spaces $x = (x_{mnk})$ is $rI-$ convergent.

### 2.15. Note

It is clear that $rI-$ limit of $x$ is not necessarily unique.

### 2.16. Definition

Consider $rI-$ limit set of $x$, which is denoted by

$$I - LIM_x^r = \left\{L \in X : x \rightarrow^{rI} L\right\},$$

then the triple sequence spaces $x = (x_{mnk})$ is said to be $rI-$ convergent if $I - LIM_x^r \neq \phi$ and $r$ is called a rough $I-$ convergence degree of $x$.

### 2.17. Definition

A triple sequence spaces $x = (x_{mnk}) \in X$ is said to be $I-$ analytic if there exists a positive real number $M$ such that

$$\left\{(m,n,k) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{mnk}|^{1/m+n+k} \geq M\right\} \in I.$$

### 2.18. Definition

A point $L \in X$ is said to be an $I-$ accumulation point of a triple sequence spaces $x = (x_{mnk})$ in a metric space $(X,d)$ if and only if for each $\varepsilon > 0$ the set

$$\{(m,n,k) \in \mathbb{N} : d\,(x_{mnk}, L) = |x_{mnk} - L| < \varepsilon\} \notin I.$$

We denote the set of all $I-$ accumulation points of $x$ by $I\,(\Gamma_x)$.

### 2.19. Definition

For a triple sequence spaces $x = (x_{mnk})$ of real numbers, the notions of ideal limit superior and ideal limit inferior are defined as follows:

$$I - limsup\ x = \left\{ \begin{array}{cc} sup\ B_x, & \text{if} B_x \neq \phi, \\ -\infty, & \text{if} B_x = \phi \end{array} \right\},$$

and

$$I - liminf\ x = \left\{ \begin{array}{cc} inf\ A_x, & \text{if} A_x \neq \phi, \\ +\infty, & \text{if} A_x = \phi \end{array} \right\},$$

where $A_x = \left\{a \in \mathbb{R} : \left\{(m,n,k) \in \mathbb{N}^3 : x_{mnk} < a\right\} \notin I\right\}$ and $B_x = \left\{b \in \mathbb{R} : \left\{(m,n,k) \in \mathbb{N}^3 : x_{mnk} > b\right\} \notin I\right\}.$

### 2.20. Definition

A triple sequence spaces $x = (x_{mnk})$ is said to be rough $I-$ convergent if $I - LIM^r x \neq \phi$. It is clear that if $I - LIM^r x \neq \phi$ for a triple sequence spaces $x = (x_{mnk})$ of real numbers, then we have $I - LIM^r x = [I - limsup\ x - r, I - liminf\ x + r]$.

### 2.21. Definition

A triple sequence spaces $x = (x_{mnk})$ of real numbers, $I - core\{x\}$ is defined to the closed interval $[I - lim\ inf\ x, I - lim\ sup\ x]$.

## 3. Main Results

### 3.1. Theorem

Let $I \subset 3^{\mathbb{N}}$ be an admimissible ideal. For a triple sequence spaces $x = (x_{mnk})$, we have diam $(I - LIM^r x_{mnk}) \leq 3r$. In general, diam $(I - LIM^r x_{mnk})$ has an upper bound.

**Proof:** Assume that diam $(LIM^r x_{mnk})$. Then, $\exists p, q, r \in LIM^r x_{mnk} \ni: |p - q - r| > 3r$. Take $\varepsilon \in \left(0, \frac{|p-q-r|}{3} - r\right)$.
Because $p, q, r \in I - LIM^r x_{mnk}$,
we have $A_1(\varepsilon) \in I, A_2(\varepsilon) \in I$ and $A_3(\varepsilon) \in I$ for every $\varepsilon > 0$, where
$A_1(\varepsilon) = \left\{(i, j, k) \in \mathbb{N}^3 : |x_{mnk} - p| \geq r + \varepsilon\right\}, A_2(\varepsilon) = \left\{(i, j, k) \in \mathbb{N}^3 : |x_{mnk} - q| \geq r + \varepsilon\right\}$,
and $A_3(\varepsilon) = \left\{(i, j, k) \in \mathbb{N}^3 : |x_{mnk} - r| \geq r + \varepsilon\right\}$.
Using the properties $F(I)$, we get

$$(A_1(\varepsilon)^c \bigcap A_2(\varepsilon)^c \bigcap A_3(\varepsilon)^c) \in F(I).$$

Thus we write, $|p - q - r| \leq |x_{mnk} - p| + |x_{mnk} - q| + |x_{mnk} - r| < (r + \varepsilon) + (r + \varepsilon) + (r + \varepsilon) < 3(r + \varepsilon)$, for all $(m, n, k) \in A_1(\varepsilon)^c \bigcap A_2(\varepsilon)^c \bigcap A_3(\varepsilon)^c$ which is a contradiction. Hence diam $(LIM^r x_{mnk}) \leq 3r$.
Now, consider a triple sequence spaces $x = (x_{mnk})$ such that $I - lim_{mnk \to \infty} x_{mnk} = L$.
Let $\varepsilon > 0$. Then we can write

$$\left\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq \varepsilon\right\} \in I$$

Thus, we have
$|x_{mnk} - p| \leq |x_{mnk} - L| + |L - p| \leq |x_{mnk} - L| + r \leq r + \varepsilon$,
for each $p \in \bar{B}_r(L) := \left\{p \in \mathbb{R}^3 : |p - L| \leq r\right\}$. Then, we get

$$|x_{mnk} - p| < r + \varepsilon$$

for each $(m, n, k) \in \left\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk} - L| < \varepsilon\right\}$. Because the triple sequence spaces $x$ is $I-$ convergent to $L$, we have

$$\left\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk} - L| < \varepsilon\right\} \in F(I).$$

Therefore, we get $p \in I - LIM^r x$. Consequently, we can write

$$I - LIM^r x = \bar{B}_r(L). \tag{1}$$

Because $diam(\bar{B}_r(L)) = 3r$, this shows that in general, the upper bound $3r$ of the diameter of the set $I - LIM^r x$ is not lower bound.

### 3.2. Theorem

Let $I \subset 3^{\mathbb{N}}$ be an admissible ideal. For an arbitrary $c \in I(\Gamma_x)$ of triple sequence spaces $x = (x_{mnk})$ we have $|L-c| \leq r$ for all $L \in I - LIM^r x$.

**Proof:** Assume on the contrary that there exist a point $c \in I(\Gamma_x)$ and $L \in I - LIM^r x$ such that $|L-c| > r$. Define $\varepsilon := \frac{|L-c|-r}{3}$. Then

$$\left\{(m,n,k) \in \mathbb{N}^3 : |L-c| < \varepsilon\right\} \subseteq \left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq r + \varepsilon\right\}. \tag{2}$$

Since $c \in I(\Gamma_x)$, we have

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - c| < \varepsilon\right\} \notin I.$$

But from definition of $I-$ convergence, since

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq r + \varepsilon\right\} \in I,$$

so by (3.2) we have

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - c| < \varepsilon\right\} \in I,$$

which contradicts the fact $c \in I(\Gamma_x)$. On the other hand, if $c \in I(\Gamma_x)$ i.e.,

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - c| < \varepsilon\right\} \notin I,$$

then

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq r + \varepsilon\right\} \notin I,$$

which contradicts the fact $L \in I - LIM^r x$.

### 3.3. Theorem

A triple sequence spaces $x = (x_{mnk}) \to^I l \Longleftrightarrow I - LIM^r x_{mnk} = \bar{B}_r(l)$.

**Proof: Necessity:** By Theorem 3.1.

**sufficiency:** Let $I - LIM^r x_{mnk} = \bar{B}_r(l)\,(\neq \phi)$. Thus the triple sequence spaces $(x_{mnk})$ is $I-$ analytic. Suppose that $x$ has another $I-$ cluster point $l'$ different from $l$. The point

$$\bar{l} = l + \frac{r}{|l-l'|}\left(l - l'\right)$$

$$\bar{l} - l' = l - l' + \frac{r}{|l-l'|}\left[\left(l-l'\right) - \left(l'-l'\right)\right]$$

$$\left|\bar{l} - l'\right| = \left|l - l'\right| + \frac{r}{|l-l'|}\left|l-l'\right|$$

$$\left|\bar{l} - l'\right| = \left|l - l'\right| + r > r.$$

Since $l' \in I(\Gamma_x)$, by Theorem 3.2, $\bar{l} \notin I - LIM^r x_{mnk}$. It is not possible as $\left|\bar{l} - l\right| = r$ and $I - LIM^r x_{mnk} = \bar{B}_r(l)$. Since $l$ is the unique-$I-$ cluster point of $x$. Hence $\Longrightarrow x_{mnk} \to^I l$.

### 3.4. Corollary

If $(X, |.,.|)$ is a strictly convex spaces and $(x_{mnk})$ is a triple sequence spaces in $X$, there exists $y_1, y_2, y_3 \in I - LIM^r x_{mnk}$ such that $|y_1 - y_2 - y_3| = 3r$, then this triple sequence $x \to^I \frac{y_1+y_2+y_3}{3}$

**Proof:** Omitted.

### 3.5. Theorem

If $I-LIM^r \neq \phi$, then $I-lim\ sup\ x_{mnk}$ and $I-lim\ inf\ x_{mnk}$ belong to the set $I-LIM^{2r}x_{mnk}$.
**Proof:** We know that $I-LIM^r x_{mnk} \neq \phi$, a triple sequence spaces $(x_{mnk})$ is $I-$ analytic. The number $I-lim\ inf x_{mnk}$ is an $I-$ cluster point of $x$ and consequently, we have

$$|l-I-lim\ inf x_{mnk}| \leq r\ \forall\ l \in I-LIM^r x.$$

Let $A = \{(m,n,k) \in \mathbb{N} : |l-x_{mnk}| \geq r+\varepsilon\}$. Now if $(m,n,k) \notin A$, then

$$|x_{mnk}-(I-lim\ inf x_{mnk})| \leq |x_{mnk}-l|+|l-(I-lim\ inf x_{mnk})| < 2r+\varepsilon.$$

Thus

$$I-lim\ inf x_{mnk} \in I-LIM^{2r}x_{mnk}.$$

Similarly it can be shown that $I-lim\ sup\ x_{mnk} \in I-LIM^{2r}x_{mnk}$.

### 3.6. Corollary

A triple sequence spaces $x=(x_{mnk})$ of real numbers. If $I-LIM^r x_{mnk} \neq \phi$, then

$$I-core\{x\} \subseteq I-LIM^{2r}x_{mnk}.$$

**Proof:** We have $I-LIM^r x_{mnk} = [I-lim\ sup\ x_{mnk}-2r, I-lim\ inf x_{mnk}+2r]$. Then the result follows from Theorem 3.5.

### 3.7. Theorem

A triple sequence spaces $x=\{x_{mnk}\}$ of real numbers. Then the $diam(I-core\{x_{mnk}\})$ of the set

$$I-core\{x_{mnk}\} = r \Longleftrightarrow I-core\{x\} = I-LIM^r x_{mnk}$$

**Proof:** $diam(I-core\{x_{mnk}\}) = r \Longleftrightarrow (I-lim\ sup\ x_{mnk})-(I-lim\ inf x_{mnk}) = r \Longleftrightarrow I-core\{x_{mnk}\} =$
$[I-lim\ inf x_{mnk}, I-lim\ sup\ x_{mnk}] =$
$[I-I-lim\ sup\ x_{mnk}-r, I-lim\ inf x_{mnk}+r] = I-LIM^r x_{mnk}$.
Also it is easy to see that
(i) $r > diam(I-core\{x_{mnk}\}) \Longleftrightarrow I-core\{x_{mnk}\} \subset I-LIM^r x_{mnk}$,
(ii) $r < diam(I-core\{x_{mnk}\}) \Longleftrightarrow I-LIM^r x_{mnk} \subset I-core\{x_{mnk}\}$.

### 3.8. Theorem

If $\bar{r} = inf\{r \geq 0 : I-LIM^r x_{mnk} \neq \phi\}$, then $\bar{r} = radius(I-core\{x_{mnk}\})$.
**Proof:** If the set $I-core\{x_{mnk}\}$ is single ton, then $radius(I-core\{x_{mnk}\}) = 0$ and the triple sequence space is $I-$ convergent, i.e., $I-LIM^0 x_{mnk} \neq \phi$. Hence we get $\bar{r} = radius(I-core\{x_{mnk}\}) = 0$.
Now assume that the set $I-core\{x_{mnk}\}$ is not a single ton. We can write $I-core\{x_{mnk}\} = [a,b]$ where $a = I-lim\ inf x_{mnk}$ and $b = I-lim\ sup\ x_{mnk}$.
Now let us assume that $\bar{r} \neq radius(I-core\{x_{mnk}\})$. If $\bar{r} < radius(I-core\{x_{mnk}\})$, then define $\bar{\varepsilon} = \frac{\frac{b-a}{2}-\bar{r}}{3}$.
Now, be definition of $\bar{r}$ implies that

$$I-LIM^{\bar{r}+\bar{\varepsilon}}x_{mnk} \neq \phi, \text{ given } \varepsilon > 0\ \exists\ l \in \mathbb{R} : A = \{(m,n,k) \in \mathbb{N} : |x_{mnk}-l| \geq (\bar{r}+\bar{\varepsilon})+\varepsilon\} \in I.$$

Since $\bar{r}+\bar{\varepsilon} < \frac{b-a}{2}$ which is a contradiction of the definition of $a$ and $b$.
If $\bar{r} > radius\left(I-core\left\{x_{mnk}\right\}\right)$, then define $\bar{\varepsilon} = \frac{\bar{r}-\frac{b-a}{2}}{3}$ and $r^{'} = \bar{r}-2\bar{\varepsilon}$. It is clear that $0 \leq r^{'} \leq \bar{r}$ and by definitions of $a$ and $b$, the number $\frac{b-a}{2} \in I-LIM^{r^{'}} x_{mnk}$. Then we get

$$\bar{r} \in \left\{r \geq 0 : I-LIM^{r} x_{mnk} \neq \phi\right\},$$

which contradicts the equality

$$\bar{r} = inf\left\{r \geq 0 : I-LIM^{r} x_{mnk} \neq \phi\right\} \text{ as } r^{'} < r.$$

### 3.9. Corollary

If a triple sequence spaces $x = (x_{mnk})$ then $I-core\left\{x_{mnk}\right\} = I-LIM^{2\bar{r}} x_{mnk}$
**Proof:** It follows that Theorem 3.7 and Theorem 3.8.

## 4. Conclusions

We introduced triple sequence spaces of rough $I-$ core. For the reference sections, consider the following introduction described the main results are motivating the research.

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