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Generalized Ostrowski Type Inequalities For Mappings Whose Second Local Fractional Derivatives Are Bounded

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Abstract. We establish two generalized Ostrowski type inequalities for functions whose second local fractional derivatives are bounded. In addition, some results of these inequalities are given.

Keywords. Ostrowski inequality, Local fractional integral, Fractal space, Hölder's inequality.

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1. Introduction

In 1938, Ostrowski established the integral inequality which is one of the fundamental inequalities of mathematics as follows (see, [12]):

Let $f:[a,b]\to\mathbb{R}$ be a differentiable mapping on (a,b) whose derivative $f^{'}:(a,b)\to\mathbb{R}$ is bounded on (a,b), i.e., $\|f'\|_\infty=\sup\limits_{t\in(a,b)}|f'(t)|<\infty$. Then, the inequality holds:

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)dt\right|\leq\left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\left\|f'\right\|_\infty \tag{1}$$

for all $x\in[a,b]$. The constant $\frac{1}{4}$ is the best possible.

Inequality (1) has wide applications in numerical analysis and in the theory of some special means; estimating error bounds for some special means, some mid-point, trapezoid and Simpson rules and quadrature rules, etc. Hence, inequality (1) has attracted considerable attention and interest from mathematicians and researchers.

In [2], the following inequality was proved by Cerone, Dragomir and Roumeliotis.

Theorem 1. Let $f:[a,b]\to\mathbb{R}$ be a twice differentiable mapping such that $f'':(a,b)\to\mathbb{R}$ is bounded on (a,b), i.e., $\|f''\|_\infty=\sup\limits_{t\in(a,b)}|f''(t)|<\infty$. Then we have the inequality

$$\begin{aligned}&\left|f(x)-\left(x-\frac{a+b}{2}\right)f'(x)-\frac{1}{b-a}\int_a^b f(t)dt\right| \qquad (2)\\ \leq\ &\left[\frac{(b-a)^2}{24}+\frac{1}{2}\left(x-\frac{a+b}{2}\right)^2\right]\left\|f''\right\|_\infty\leq\frac{(b-a)^2}{6}\left\|f''\right\|_\infty\end{aligned}$$

for all $x\in[a,b]$.

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In [5], Dragomir and Barnett proved the following inequalities.

Theorem 2. Let $f : [a,b] \to \mathbb{R}$ be a twice differentiable mapping such that $f'' : (a,b) \to \mathbb{R}$ is bounded on (a,b), i.e., $\|f''\|_\infty = \sup_{t\in(a,b)} |f''(t)| < \infty$. Then we have the inequality

$$\begin{aligned} & \left| f(x) - \frac{f(b)-f(a)}{b-a}\left(x-\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(t)dt \right| \\ \leq \quad & \frac{(b-a)^2}{2}\left\{\left[\left(\frac{x-\frac{a+b}{2}}{b-a}\right)^2+\frac{1}{4}\right]+\frac{1}{12}\right\}\|f''\|_\infty \\ \leq \quad & \frac{(b-a)^2}{6}\|f''\|_\infty \end{aligned}$$

for all $x \in [a,b]$.

In recent years, researchers have studied Qstrowski type inequalities for convex functions and mappings whose derivatives are bounded. You can check ([2], [4], [5]-[10], [13]-[15], [22]) and the references included there.

2. Preliminaries

Recall the set R^α of real line numbers and use the Gao-Yang-Kang's idea to describe the definition of the local fractional derivative and local fractional integral, see [17, 18] and so on.

Recently, the theory of Yang's fractional sets [17] was introduced as follows.

For $0 < \alpha \leq 1$, we have the following α-type set of element sets:

Z^α : The α-type set of integer is defined as the set $\{0^\alpha, \pm 1^\alpha, \pm 2^\alpha, ..., \pm n^\alpha, ...\}$.

Q^α : The α-type set of the rational numbers is defined as the set $\{m^\alpha = \left(\frac{p}{q}\right)^\alpha : p,q \in Z,\ q \neq 0\}$.

J^α : The α-type set of the irrational numbers is defined as the set $\{m^\alpha \neq \left(\frac{p}{q}\right)^\alpha : p,q \in Z,\ q \neq 0\}$.

R^α : The α-type set of the real line numbers is defined as the set $R^\alpha = Q^\alpha \cup J^\alpha$.

If a^α, b^α and c^α belongs the set R^α of real line numbers, then

(1) $a^\alpha + b^\alpha$ and $a^\alpha b^\alpha$ belongs the set R^α;

(2) $a^\alpha + b^\alpha = b^\alpha + a^\alpha = (a+b)^\alpha = (b+a)^\alpha$;

(3) $a^\alpha + (b^\alpha + c^\alpha) = (a+b)^\alpha + c^\alpha$;

(4) $a^\alpha b^\alpha = b^\alpha a^\alpha = (ab)^\alpha = (ba)^\alpha$;

(5) $a^\alpha (b^\alpha c^\alpha) = (a^\alpha b^\alpha) c^\alpha$;

(6) $a^\alpha (b^\alpha + c^\alpha) = a^\alpha b^\alpha + a^\alpha c^\alpha$;

(7) $a^\alpha + 0^\alpha = 0^\alpha + a^\alpha = a^\alpha$ and $a^\alpha 1^\alpha = 1^\alpha a^\alpha = a^\alpha$.

The definition of the local fractional derivative and local fractional integral can be given as follows.

Definition 1. [17] A non-differentiable function $f: R \to R^{\alpha}$, $x \to f(x)$ is called to be local fractional continuous at x_0, if for any $\varepsilon > 0$, there exists $\delta > 0$, such that

$$|f(x) - f(x_0)| < \varepsilon^{\alpha}$$

holds for $|x - x_0| < \delta$, where $\varepsilon, \delta \in R$. If $f(x)$ is local continuous on the interval (a,b), we denote $f(x) \in C_{\alpha}(a,b)$.

Definition 2. [17] The local fractional derivative of $f(x)$ of order α at $x = x_0$ is defined by

$$f^{(\alpha)}(x_0) = \left. \frac{d^{\alpha} f(x)}{dx^{\alpha}} \right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\alpha}},$$

where $\Delta^{\alpha}(f(x) - f(x_0)) \cong \Gamma(\alpha+1)(f(x) - f(x_0))$.

If there exists $f^{(k+1)\alpha}(x) = \overbrace{D_x^{\alpha} ... D_x^{\alpha}}^{k+1 \text{ times}} f(x)$ for any $x \in I \subseteq R$, then we denoted $f \in D_{(k+1)\alpha}(I)$, where $k = 0, 1, 2, ...$

Definition 3. [17] Let $f(x) \in C_{\alpha}[a,b]$. Then the local fractional integral is defined by,

$${}_aI_b^{\alpha} f(x) = \frac{1}{\Gamma(\alpha+1)} \int_a^b f(t)(dt)^{\alpha} = \frac{1}{\Gamma(\alpha+1)} \lim_{\Delta t \to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^{\alpha},$$

with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_1, \Delta t_2, ..., \Delta t_{N-1}\}$, where $[t_j, t_{j+1}]$, $j = 0, ..., N-1$ and $a = t_0 < t_1 < ... < t_{N-1} < t_N = b$ is partition of interval $[a,b]$.

Here, it follows that ${}_aI_b^{\alpha} f(x) = 0$ if $a = b$ and ${}_aI_b^{\alpha} f(x) = -{}_bI_a^{\alpha} f(x)$ if $a < b$. If for any $x \in [a,b]$, there exists ${}_aI_x^{\alpha} f(x)$, then we denoted by $f(x) \in I_x^{\alpha}[a,b]$.

Lemma 1. [17]

(1) (Local fractional integration is anti-differentiation) Suppose that $f(x) = g^{(\alpha)}(x) \in C_{\alpha}[a,b]$, then we have

$${}_aI_b^{\alpha} f(x) = g(b) - g(a).$$

(2) (Local fractional integration by parts) Suppose that $f(x), g(x) \in D_{\alpha}[a,b]$ and $f^{(\alpha)}(x), g^{(\alpha)}(x) \in C_{\alpha}[a,b]$, then we have

$${}_aI_b^{\alpha} f(x) g^{(\alpha)}(x) = f(x)g(x)|_a^b - {}_aI_b^{\alpha} f^{(\alpha)}(x) g(x).$$

Lemma 2. [17] We have

i) $\dfrac{d^{\alpha} x^{k\alpha}}{dx^{\alpha}} = \dfrac{\Gamma(1+k\alpha)}{\Gamma(1+(k-1)\alpha)} x^{(k-1)\alpha}$;

ii) $\dfrac{1}{\Gamma(\alpha+1)} \int_a^b x^{k\alpha}(dx)^{\alpha} = \dfrac{\Gamma(1+k\alpha)}{\Gamma(1+(k+1)\alpha)} \left(b^{(k+1)\alpha} - a^{(k+1)\alpha}\right)$, $k \in R$.

Lemma 3 (Generalized Hölder's inequality)**.** [17] Let $f, g \in C_{\alpha}[a,b]$, $p, q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$, then

$$\frac{1}{\Gamma(\alpha+1)} \int_a^b |f(x)g(x)| (dx)^{\alpha} \leq \left(\frac{1}{\Gamma(\alpha+1)} \int_a^b |f(x)|^p (dx)^{\alpha} \right)^{\frac{1}{p}} \left(\frac{1}{\Gamma(\alpha+1)} \int_a^b |g(x)|^q (dx)^{\alpha} \right)^{\frac{1}{q}}.$$

For more information and recent developments on local fractional theory, please refer to [1], [3], [11], [16]-[21].

In this study, it is established two inequalities that are connected with the celebrated generalized Ostrowski type inequalities using functions whose second local fractional derivatives are bounded.

3. Main Results

In order to prove our main results we need the following lemma:

Lemma 4. Let $I \subseteq R$ be an interval, $f : I^0 \subseteq R \to R^{\alpha}$ (I^0 is the interior of I) such that $f \in D_{2\alpha}(I^0)$ and $f^{(2\alpha)} \in C_{2\alpha}[a,b]$ for $a,b \in I^0$ with $a < b$. Then the following identity holds:

$$\begin{aligned} & \frac{1}{2^{\alpha}(b-a)^{\alpha}} \frac{1}{\Gamma(1+\alpha)} \int_a^b P_h(x,t;\alpha) f^{(2\alpha)}(t)(dt)^{\alpha} \qquad (3) \\ = & \frac{(h-2)^{\alpha}}{2^{\alpha}} \left(x - \frac{a+b}{2}\right)^{\alpha} f^{(\alpha)}(x) + \frac{\Gamma(1+2\alpha)}{2^{\alpha}\Gamma(1+\alpha)} f(x) \\ & - \frac{\Gamma(1+\alpha)}{(b-a)^{\alpha}} (m_h(x))^{\alpha} \left[\frac{f(b)-f(a)}{2^{\alpha}}\right] \\ & - \frac{1}{2^{\alpha}(b-a)^{\alpha}} \frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^{\alpha} \\ = & : S_{x,h}(f;\alpha) \end{aligned}$$

for

$$P_h(x,t;\alpha) := \begin{cases} (a-t)^{\alpha}(t-a-m_h(x))^{\alpha} & ,a \le t < x \\ (b-t)^{\alpha}(t-b-m_h(x))^{\alpha} & ,x \le t \le b \end{cases}$$

where $m_h(x) = h\left(x - \frac{a+b}{2}\right)$, $h \in (0,2]$ and $x \in [a,b]$.

Proof. By definition of function $P_h(x,t;\alpha)$, we find that

$$\begin{aligned} & \frac{1}{\Gamma(1+\alpha)} \int_a^b P_h(x,t;\alpha) f^{(2\alpha)}(t)(dt)^{\alpha} \\ = & \frac{1}{\Gamma(1+\alpha)} \int_a^x (a-t)^{\alpha}(t-a-m_h(x))^{\alpha} f^{(2\alpha)}(t)(dt)^{\alpha} \\ & + \frac{1}{\Gamma(1+\alpha)} \int_x^b (b-t)^{\alpha}(t-b-m_h(x))^{\alpha} f^{(2\alpha)}(t)(dt)^{\alpha}. \end{aligned}$$

Applying local fractional integration by parts and using the Lemma 2, we obtain

$$\begin{aligned}
&\frac{1}{\Gamma(1+\alpha)}\int_a^b P_h(x,t;\alpha) f^{(2\alpha)}(t)(dt)^\alpha \\
= \;& (h-2)^\alpha (b-a)^\alpha \left(x-\frac{a+b}{2}\right)^\alpha f^{(\alpha)}(x) \\
&+\frac{1}{\Gamma(1+\alpha)}\int_a^x \left[\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}(t-a)^\alpha - \Gamma(1+\alpha)(m_h(x))^\alpha\right] f^{(\alpha)}(t)(dt)^\alpha \\
&+\frac{1}{\Gamma(1+\alpha)}\int_x^b \left[\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}(t-b)^\alpha - \Gamma(1+\alpha)(m_h(x))^\alpha\right] f^{(\alpha)}(t)(dt)^\alpha .
\end{aligned}$$

If we apply local fractional integration by parts again, then we obtain desired equality (3). ■

Now, we deduce some new inequalities involving local fractional integrals and also give some results related to these inequalities.

Theorem 3. The assumptions of Lemma 4 are satisfied. If $f^{(2\alpha)}$ is bounded on (a,b), then we have the inequalities

$$\begin{aligned}
&\left|S_{x,h}(f;\alpha)\right| \\
\leq\;& \left\{\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\left(\frac{(b-x)^{3\alpha}+(x-a)^{3\alpha}}{2^\alpha (b-a)^\alpha}\right) - h^\alpha \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left(x-\frac{a+b}{2}\right)^\alpha \right. \\
&\left. -\frac{1}{(b-a)^\alpha}\left[\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\right][m_h(x)]^{3\alpha}\right\}\left\|f^{(2\alpha)}\right\|_\infty
\end{aligned} \tag{4}$$

for all $a \leq x \leq \frac{a+b}{2}$ with $h \in (0,2]$ and

$$\begin{aligned}
&\left|S_{x,h}(f;\alpha)\right| \\
\leq\;& \left\{\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\left(\frac{(b-x)^{3\alpha}+(x-a)^{3\alpha}}{2^\alpha (b-a)^\alpha}\right) - h^\alpha \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left(x-\frac{a+b}{2}\right)^\alpha \right. \\
&\left. +\frac{1}{(b-a)^\alpha}\left[\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\right][m_h(x)]^{3\alpha}\right\}\left\|f^{(2\alpha)}\right\|_\infty
\end{aligned} \tag{5}$$

for all $\frac{a+b}{2} < x \leq b$ with $h \in (0,2]$, where $m_h(x) = h\left(x-\frac{a+b}{2}\right)$.

Proof. We take absolute value of (3). Using bounded of the mapping $f^{(2\alpha)}$, we find that

$$\begin{aligned} & \left|S_{x,h}(f;\alpha)\right| \qquad (6) \\ \leq\ & \frac{1}{2^{\alpha}(b-a)^{\alpha}}\frac{1}{\Gamma(1+\alpha)}\int_a^b |P_h(x,t;\alpha)|\left|f^{(2\alpha)}(t)\right|(dt)^{\alpha} \\ \leq\ & \frac{\left\|f^{(2\alpha)}\right\|_{\infty}}{2^{\alpha}(b-a)^{\alpha}}\left[\frac{1}{\Gamma(1+\alpha)}\int_a^x |a-t|^{\alpha}|t-a-m_h(x)|^{\alpha}(dt)^{\alpha}\right. \\ & \left.+\frac{1}{\Gamma(1+\alpha)}\int_x^b |b-t|^{\alpha}|t-b-m_h(x)|^{\alpha}(dt)^{\alpha}\right]. \end{aligned}$$

Now, let us observe that

$$\begin{aligned} & \frac{1}{\Gamma(1+\alpha)}\int_p^r |t-p|^{\alpha}|t-q|^{\alpha}(dt)^{\alpha} \qquad (7) \\ =\ & \frac{1}{\Gamma(1+\alpha)}\int_p^q (t-p)^{\alpha}(q-t)^{\alpha}(dt)^{\alpha}+\frac{1}{\Gamma(1+\alpha)}\int_q^r (t-p)^{\alpha}(t-q)^{\alpha}(dt)^{\alpha} \\ =\ & 2^{\alpha}\left[\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\right](q-p)^{3\alpha} \\ & +\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}(r-p)^{3\alpha}-\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}(q-p)^{\alpha}(r-p)^{2\alpha} \end{aligned}$$

for all r,p,q such that $p\leq q\leq r$.

It should be calculated calculate the above local fractional integrals for the cases $a\leq x\leq \frac{a+b}{2}$ and $\frac{a+b}{2}<x\leq b$;

For all $a\leq x\leq \frac{a+b}{2}$, we observe that

$$\begin{aligned} & \frac{1}{\Gamma(1+\alpha)}\int_a^x |a-t|^{\alpha}|t-a-m_h(x)|^{\alpha}(dt)^{\alpha} \qquad (8) \\ =\ & \frac{1}{\Gamma(1+\alpha)}\int_a^x (t-a)^{\alpha}(t-a-m_h(x))^{\alpha}(dt)^{\alpha} \\ =\ & \frac{1}{\Gamma(1+\alpha)}\int_0^{x-a} u^{\alpha}(u-m_h(x))^{\alpha}(du)^{\alpha} \\ =\ & \frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}(x-a)^{3\alpha}-\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}[m_h(x)]^{\alpha}(x-a)^{2\alpha} \end{aligned}$$

and using the equality (7), we obtain

$$\begin{aligned}
&\frac{1}{\Gamma(1+\alpha)}\int_x^b |b-t|^\alpha |t-b+m_h(x)|^\alpha (dt)^\alpha \qquad (9)\\
=\ & -2^\alpha\left[\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\right][m_h(x)]^{3\alpha}\\
&+\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}(b-x)^{3\alpha}+\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}[m_h(x)]^\alpha (b-x)^{2\alpha}.
\end{aligned}$$

Substituting the equalities (8) and (9) in (6), we get the inequality (4).
For all $\frac{a+b}{2}<x\leq b$; if we use the equality (7), we obtain

$$\begin{aligned}
&\frac{1}{\Gamma(1+\alpha)}\int_a^x |a-t|^\alpha |t-a-m_h(x)|^\alpha (dt)^\alpha \qquad (10)\\
=\ & 2^\alpha\left[\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\right][m_h(x)]^{3\alpha}\\
&+\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}(x-a)^{3\alpha}-\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}[m_h(x)]^\alpha (x-a)^{2\alpha}
\end{aligned}$$

and we observe that

$$\begin{aligned}
&\frac{1}{\Gamma(1+\alpha)}\int_x^b |b-t|^\alpha |t-b+m_h(x)|^\alpha (dt)^\alpha \qquad (11)\\
=\ & \frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}(b-x)^{3\alpha}+\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}[m_h(x)]^\alpha (b-x)^{2\alpha}.
\end{aligned}$$

Substituting the equalities (10) and (11) in (6), we obtain the inequality (5). Hence, the proof is completed.
■

Corollary 1. If we choose $h=0$ in Theorem 3, then we have

$$\begin{aligned}
=\ & \left|\frac{\Gamma(1+2\alpha)}{2^\alpha\Gamma(1+\alpha)}f(x)-\left(x-\frac{a+b}{2}\right)^\alpha f^{(\alpha)}(x)\right.\\
&\left.-\frac{1}{2^\alpha(b-a)^\alpha}\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^\alpha\right|\\
&\leq\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\left(\frac{(b-x)^{3\alpha}+(x-a)^{3\alpha}}{2^\alpha(b-a)^\alpha}\right)\left\|f^{(2\alpha)}\right\|_\infty.
\end{aligned}$$

Corollary 2. If we take $h=0$ in Theorem 3, then we have the inequality

$$\begin{aligned}
&\left|\frac{\Gamma(1+2\alpha)}{2^\alpha\Gamma(1+\alpha)}f\left(\frac{a+b}{2}\right)-\frac{1}{2^\alpha(b-a)^\alpha}\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^\alpha\right|\\
\leq\ & \frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\left(\frac{(b-a)^{2\alpha}}{8^\alpha}\right)\left\|f^{(2\alpha)}\right\|_\infty.
\end{aligned}$$

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